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# Tachyon condensation using the disc partition function

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Abstract: It has been recently proposed that the background independent open superstring field theory action is given by the disc partition function with all possible open string operators inserted at the boundary of the disc. We use this proposal to study tachyon condensation in the D0-D2 system. We evaluate the disc partition function for the D0-D2 system in presence of a large Neveu-Schwarz B-field using perturbation theory. This perturbative expansion of the disc partition function makes sense as the boundary tachyon operator for the large Neveu-Schwarz B-field is almost marginal. We find that the mass defect for the formation of the D0-D2 bound state agrees exactly with the expected result in the large B-field limit.

Keywords: Superstrings and Heterotic Strings, D-branes, Conformal Field Models in String Theory.

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## Contents



# 1. Introduction

Recent works have addressed the study of open string tachyon condensation from various point of view. In this paper we focus on the string field theoretic approach to study tachyon condensation. The open string tachyon in bosonic string theory signals the instablity of the D25-brane to decay to the vacuum. There exists a a stationary point in the tachyon potential [1, 2]. The value of the tachyon potential at this stationary point should agree with the tension of the unstable brane. This has been verified to a remarkable degree of accuracy in cubic string field theory using level truncation [1] and [3]–[6]. The physics of the solitons in the tachyon potential also seem to be reproduced in the level truncation approximation scheme [13, 14, 15]. More recently, the exact tachyon potential has been calculated using the background independent open string field theory formulated by [7, 8]. The stationary point of the tachyon potential from background independent open string field theory agrees precisely with the tension of the D-brane [9, 10, 11]. Condensation to lower dimensional branes also reproduced their expected tensions.

There is a similar story for the case of unstable branes and D-brane anti-D-brane systems of superstring field theory. For these systems the tachyon potential has been evaluated using level truncation in the superstring theory formulated by Berkovits [12]. The stationary point of this potential agrees with the tension of the branes to a high degree of accuracy [16]–[19]. It has been recently proposed in [20] that the The background independent superstring field theory action is the disc partition function with all possible operators inserted on the boundary of the disc. Precisely the boundary string field theory action is given by

$$S[\lambda_i] = Z[\lambda_i]\,, \tag{1.1}$$

where $Z[\lambda_i]$ is the disc partition function of the world sheet conformal field theory. $\lambda_i$ are the couplings of the various boundary operators. This is a generalization of the proposal of [21, 22] which restricted the operators to those corresponding to the massless modes (see also [23]). Using this proposal tachyon condensation on unstable D-branes of type II sting theory was studied in [20]. The tachyon potential found had a minimum which agreed with the tension of the unstable D-brane. Condensation to lower dimensional branes reproduced their expected tensions.[1]

There are open string tachyons in other systems. Two examples of such systems are The D0-D4 system with a Neveu-Schwarz B-field in the spatial directions of the D4-brane [32]. and the D0-D2 system. The tachyon potential for the D0-D2 system was studied in first quantized string theory in [33]. It is interesting to study tachyon condensation in these systems for the following reasons. These unstable systems decay to BPS states unlike the unstable D-branes which decay to the vacuum. The tachyon potential for these systems lies outside the universality class of the unstable D-branes [34]. In fact they have a parameter , the B-field which offers a controlled expansion in the study of the tachyon potential. This parameter allows the interpolation between these systems and the D-brane anti-D-brane system. Study of tachyon condensation in these systems offer tests to proposals of superstring field theory action. This is important because the formulation of superstring field theory is not complete.

In this paper we will use the the disc partition function to study tachyon condensation in the D0-D2 system. In general the disc partition function with all possible boundary operators is not well defined. For unstable D-branes it was possible to evaluate the disc partition function exactly as the tachyon boundary operator was

[1]Exact results for tachyon condensation has also been obtain by introducing noncommutativity [24]–[28]. Recently Matrix theory has been used to study tachyon condensation [29, 30, 31]

linear and the theory was free. It was also argued that this operator does not mix with any others in renormalization group flow. For the D0-D2 system the tachyon operator is a twist operator. We do not have the facility of a free theory. But we have the facility of making this operator almost marginal by introducing a B-field in the spatial direction of the D2-brane. For large values of the B-field the tachyon operator becomes almost marginal. Then we can set up a well defined perturbative expansion of the disc partition function in the coupling $\lambda$.[2] We show that the leading order term in the mass defect obtained from the disc partition function agrees exactly with the expected mass defect from the BPS formula. This lends support to the proposal that the disc partition function is the background independent superstring field theory action.

The organization of this paper is as follows. In section 2 we review the formulation of how to introduce boundary interaction in the disc partition function in a manifestly world sheet supersymmetric manner [36, 37, 20]. These interactions correspond to boundary tachyonic interaction. We also review the point splitting regularization introduced by [20]. In section 3 we introduce the D0-D2 system with the B-field and construct the boundary tachyon operators. We evaluate the various correlation functions of these operators required to evaluate the tachyon potential. In section 4 we use all the ingredients to evaluate the disc partition function till the quartic order in couplings. In section 5 we compare the mass defect obtained from the disc partition function to the leading order from the BPS formula. We obtain exact agreement. Section 6 contains our conclusions. The appendices A, B and C contain details regarding the tachyon operator and the various correlation functions.

## 2. The disc partition function in perturbation theory

The disc partition function for the world sheet of the Neveu-Schwarz superstring is given by

$$Z[\lambda_i] = \int [d\psi^\mu][dX^\mu] e^{-S_{\rm Bulk} - S_{\rm Boundary}}, \tag{2.1}$$

where

$$S_{\rm Bulk} = \frac{1}{4\pi}\int d^2 z \left(\partial X^\mu \bar\partial X_\mu + \psi^\mu \bar\partial \psi_\mu + \tilde\psi^\mu \partial \tilde\psi_\mu\right). \tag{2.2}$$

Here $\mu$ runs from $0, \ldots, 9$ and the signature of the world sheet and the space time is euclidean. We have set $\alpha' = 2$, the integral in $S_{\rm Bulk}$ is over a disc of radius 1. The disc partition function depends on $\lambda_i$ which stands for the various couplings in the boundary interaction. We introduce the boundary interaction in $S_{\rm Boundary}$ preserving $N = 1$ supersmmetry. The boundary superspace coordinates are $(\tau, \theta)$

[2]The disc partition function with the B-field turned on has been evaluated in [35] for the bosonic case.

with $\tau \in [-\pi, \pi]$ and $\theta$ the boundary Grassman coordinate. The boundary action in this superfield notation is given by

$$S_{\text{Boundary}} = -\int \frac{d\tau}{2\pi} d\theta \left( \Gamma D \Gamma + (O_1 + \theta O_2)\Gamma \right), \tag{2.3}$$

here

$$D = \partial_\theta + \theta \partial_\tau \qquad \Gamma = \eta + \theta F. \tag{2.4}$$

$\Gamma$ is the fermionic degree of freedom living on the boundary. $O_1$ and $O_2$ are the components of the superfield corresponding to the tachyonic boundary interaction. The integral in (2.3) is over the boundary of the disc and runs from $-\pi$ to $\pi$. Eliminating $\Gamma$ by using its equation of motion and integration over $\theta$ we obtain the following boundary action.

$$S_{\text{Boundary}} = \frac{1}{4} \int \frac{d\tau}{2\pi} \left( O_1^2(\tau) + O_2(\tau) \int \frac{1}{2} \Theta(\tau - \tau') O_2(\tau') d\tau' \right), \tag{2.5}$$

where $\Theta(\tau) = 1$ for $\tau > 0$ and $\Theta(\tau) = -1$ for $\tau < 0$. We also have

$$\frac{d}{d\tau} \Theta(\tau) = 2\delta(\tau). \tag{2.6}$$

Though the boundary interaction is non-local it is well defined in the Neveu-Schwarz sector as $\eta$ and $\psi$ do not have zero modes in this sector. We formulate the perturbative expansion of the disc partition function by simply expanding in powers of the boundary interaction. We have the following expansion.

$$\begin{aligned} \frac{Z[\lambda_i]}{Z[0]} &= 1 - \langle S_{\text{Boundary}} \rangle + \frac{1}{2} \langle (S_{\text{Boundary}})^2 \rangle + \cdots \\ &= 1 + S_2 + S_4 + \cdots . \end{aligned} \tag{2.7}$$

We will justify this expansion later for boundary interactions which are almost marginal. We will be interested only till the quartic term in the expansion of the disc partition function in (2.7).

To evaluate each term in the expansion in (2.7) we have to use a renormalization prescription. We use a point splitting renormalization prescription introduced by [20]. For the term $S_2$ in (2.7) it is given by

$$S_2 = \lim_{\epsilon \to 0} -\frac{1}{4} \left( \int \frac{d\tau}{2\pi} \left\langle O_1(\tau) O_1(\tau - \epsilon) + O_2(\tau) \int \frac{1}{2} \Theta(\tau - \tau' - \epsilon) O_2(\tau') d\tau' \right\rangle \right). \tag{2.8}$$

It is easy to see that this renormalization prescription preserves world sheet supersymmetry as $S_2$ is $\frac{1}{2} \int d\tau d\theta \langle (O_1(\tau) + \theta O_2(\tau)) \Gamma(\tau - \epsilon, \theta) \rangle$. Carrying over this prescription to the quartic term $S_4$ we get

$$S_4 = \frac{1}{8} \lim_{\epsilon_1 \to 0, \epsilon_2 \to 0} \int d\tau_1 d\tau_2 d\theta_1 d\theta_2 \langle (O_1 + \theta_1 O_2) \Gamma(\tau_1 - \epsilon_1, \theta_1) (O_1 + \theta_2 O_2) \Gamma(\tau_1 - \epsilon_2, \theta_2) \rangle. \tag{2.9}$$

To simplify the calculations we take the limit in (2.9) along the 45° line in the $\epsilon_1, \epsilon_2$ plane. Therefore to evaluate the quartic term in the expansion of the disc partition function we use the prescription

$$S_4 = \frac{1}{8} \lim_{\epsilon \to 0} \int d\tau_1 d\tau_2 d\theta_1 d\theta_2 \langle (O_1 + \theta_1 O_2) \Gamma(\tau_1 - \epsilon, \theta_1)(O_1 + \theta_2 O_2) \Gamma(\tau_1 - \epsilon, \theta_2) \rangle \,. \quad (2.10)$$

## 3. The tachyon in the D0-D2 system

In this section we review the D0-D2 system with a Neveu-Schwarz B-field along the spatial directions of the D2-brane. See [38] for a discussion of the D$p$-D$p'$ system with the Neveu-Schwarz B-field. We then construct the tachyon operators $O_1$ and $O_2$ for this system.

### 3.1 The D0-D2 system

Consider a single D0-brane and a single D2-brane of type IIA string theory in ten dimensions configured as follows. The D2-brane is extended along the directions $x^1$ and $x^2$. The open string spectrum consists of excitations of strings joining the D0-brane with itself and the D2-brane with itself. Then there are excitations of the open string joining the D0-brane with the D2-brane. We denote the open strings joining the D0-brane with the D2-brane as the $(0,2)$ strings. The lowest excitation of the $(0,2)$ strings is a tachyon. Let the string world sheet coordinates of the $(0,2)$ string be $X^\mu(\sigma^0, \sigma^1)$.$\mu$ runs form $0, \ldots, 9$, and $\sigma^1$ lies between 0 and $\pi$. We work with euclidean world sheet signature. Now turn on a constant B-field along the spatial directions of the D2-brane. This changes the mass of the $(0,2)$ tachyon. The B-field is given by

$$B_{ij} = \frac{1}{4\pi} \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}, \quad (3.1)$$

where $i, j \in \{1, 2\}$. We choose the metric $g_{ij} = \delta_{ij}$. The boundary conditions of the world sheet coordinates with these moduli turned on is given by

$$\begin{aligned} \partial_{\sigma^1} X^i + 4\pi i B_{ij} \partial_{\sigma^0} X^j |_{\sigma^1 = \pi} &= 0 \\ \partial_{\sigma^0} X^i |_{\sigma^1 = 0} &= 0 \\ \partial_{\sigma^0} X^a |_{\sigma^1 = 0, \sigma^1 = \pi} &= 0 \qquad \text{where } a = 3, \ldots, 9 \\ \partial_{\sigma^1} X^0 |_{\sigma^1 = 0, \sigma^1 = \pi} &= 0 \,. \end{aligned} \quad (3.2)$$

The non-trivial mode expansions arise for the world sheet coordinates-ordinates $X^i$, It is convenient to define coordinates-ordinates

$$X^+ = X^1 + iX^2 \qquad X^- = X^1 - iX^2 \,. \quad (3.3)$$

The mode expansions of $X^+$ and $X^-$ are given by

$$X^+ = i\sqrt{2}\sum_n \left[\frac{\alpha^+_{n-\nu}}{n-\nu}e^{-(n-\nu)(\sigma^0+i\sigma^1)} - \frac{\alpha^+_{n-\nu}}{n-\nu}e^{-(n-\nu)(\sigma^0-i\sigma^1)}\right]$$
$$X^- = i\sqrt{2}\sum_n \left[\frac{\alpha^-_{n+\nu}}{n+\nu}e^{-(n+\nu)(\sigma^0+i\sigma^1)} - \frac{\alpha^-_{n+\nu}}{n+\nu}e^{-(n+\nu)(\sigma^0-i\sigma^1)}\right]. \tag{3.4}$$

Where

$$e^{2\pi i\nu} = -\frac{1+ib}{1-ib}, \qquad 0 \le \nu < 1 \tag{3.5}$$

The only non zero commutation relations are

$$[\alpha^+_{n-\nu}, \alpha^-_{m+\nu}] = (n-\nu)\delta(n+m). \tag{3.6}$$

The mode expansions of world sheet superpartners of the bosonic fields are fixed by supersymmetry. The mode expansions of $\psi^+$ and $\bar{\psi}^+$ of $X^+$ is given by

$$\psi^+ = -i\sqrt{2}\sum_n \psi^+_{n+1/2-\nu}e^{-(n+1/2-\nu)(\sigma^0+i\sigma^1)}$$
$$\bar{\psi}^+ = i\sqrt{2}\sum_n \psi^+_{n+1/2-\nu}e^{-(n+1/2-\nu)(\sigma^0-i\sigma^1)}. \tag{3.7}$$

The mode expansions of the superpartners of $X^-$ are given by

$$\psi^- = -i\sqrt{2}\sum_n \psi^+_{n+1/2+\nu}e^{-(n+1/2+\nu)(\sigma^0+i\sigma^1)}$$
$$\bar{\psi}^+ = i\sqrt{2}\sum_n \psi^+_{n+1/2+\nu}e^{-(n+1/2+\nu)(\sigma^0-i\sigma^1)}. \tag{3.8}$$

We have written the mode expansions in the Neveu-Schwarz sector. The only non-zero anti-commutation rules are given by

$$\{\psi^+_{n+1/2-\nu}, \psi^-_{m+1/2+\nu}\} = \delta(m+n). \tag{3.9}$$

The $(\text{mass})^2$ of tachyon in the $(0,2)$ strings is determined from the zero point energy. The zero point energy for two of the lowest states in the Neveu-Schwarz sector given by

$$E_0 = -\frac{1}{2} + \frac{\nu}{2} \qquad E_+ = -\frac{\nu}{2}. \tag{3.10}$$

The state with energy $E_0$ is retained by the GSO projection. We see that the $(\text{mass})^2$ of the tachyon decreases as $b \to \infty$. As $b \to \infty$ we have $\nu \to 1 - 1/(\pi b)$. Thus the zero point energy in the limit of large $b$ is given by $E_0 = -1/\pi b$. For $b \to -\infty$ the mode expansions in (3.4), (3.7) and (3.8) reduce to that of Dirichlet-Dirichlet boundary conditions. The induced D0-brane charge on the D2-brane is proportional to $b$. Therefore for $b \to -\infty$ the system reduces to the D0-brane anti-D0-brane system. In fact in this limit the zero point energy $E_0 = -1/2$ which is that of the tachyon of the D0-brane anti-D0-brane system.

### 3.2 The calculation of the mass defect

The tachyon in the $(0,2)$ strings signal the instability to form a D0-D2 bound state. In this section we will use BPS mass formulae to evaluate the mass defect in the formation of the bound state. The mass of the D0-brane is given by[3]

$$M_{D0} = \frac{1}{g\sqrt{2}} \,. \tag{3.11}$$

The Mass of the D2-brane with the value of the B-field in the spatial direction of the D2-brane is given by

$$M_{D2} = \frac{1}{g\sqrt{2}}\sqrt{1+b^2} \,. \tag{3.12}$$

This can be easily understood from the Dirac-Born-Infeld action of the D2-brane. We have compactified the D2-brane on a square torus of radius $\sqrt{2}$ for simplicity, the mass defect is independent of this compactification. The BPS mass formula of the bound state of the D0-brane within the D2-brane is given by

$$M^2 = \frac{1}{2g^2}(Q_0 + bQ_2)^2 + \frac{1}{2g^2}Q_2^2 \,, \tag{3.13}$$

where $Q_0$ and $Q_2$ are the number of D0-branes and D2-branes respectively. Substituting the $Q_0 = Q_2 = 1$ we obtain

$$M_{D0-D2} = \frac{1}{g\sqrt{2}}\sqrt{1+(1+b)^2} \,. \tag{3.14}$$

The mass defect for the formation of the D0-D2 bound state is given by

$$\Delta M = M_{D0-D2} - (M_{D0} + M_{D2}) \,. \tag{3.15}$$

We require the leading order term in the mass defect for $b \to \infty$. This is given by

$$\Delta M = -\frac{1}{2\sqrt{2}gb^2} \,. \tag{3.16}$$

Note that this is the mass defect obtained in [28] for the formation of the D0-D2 bound state. From the discussion in section 3.1 we note that the system reduces to the D0-brane anti-D0-brane system for $b \to -\infty$. This also reflected in the mass defect. We find that for $b \to -\infty$ the mass defect $\Delta M = -\sqrt{2}/g$, which is twice the mass of the D0-brane.

## 4. The tachyon operators and their correlation functions

In this section we construct the boundary operators $O_1$ and $O_2$ corresponding to the tachyon of the D0/D2 system. Then we write down the correlation functions of these operators needed for the evaluation of the disc partition function.

[3]Note that we have used $\alpha' = 2$.



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### 4.1 The tachyon operators

The Hilbert space of the D0/D2 system can be represented using a $2 \times 2$ Matrix.

$$\begin{pmatrix} (0,0)\text{Strings} & (0,2)\text{Strings} \\ (2,0)\text{Strings} & (2,2)\text{Strings} \end{pmatrix} . \tag{4.1}$$

The operator $O_1$ corresponding to the $(0,2)$ tachyon should involve twist fields which change the boundary conditions. This is clear from the fact that the $(0,2)$ strings have are fractionally moded. We can see this in the mode expansions in (3.4), (3.7) and (3.8). Using these conditions the boundary operator $O_1$ is given by

$$O_1 = \lambda_+ \Sigma_\nu \sigma_+ + \lambda_- \Sigma_{-\nu} \sigma_- \,, \tag{4.2}$$

where

$$\Sigma_\nu = \sigma_\nu e^{i\nu H} \quad \Sigma_{-\nu} = \sigma_{-\nu} e^{-i\nu H} \,. \tag{4.3}$$

The twist operators $\sigma_\nu, \sigma_{-\nu}$ and $H$ are defined in appendix A (A.1) and (A.2). $\sigma_+$ and $\sigma_-$ are given by

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} . \tag{4.4}$$

The operator $O_1$ does not have diagonal entries as expected. The twist operator $\Sigma_\nu$ comes with the matrix $\sigma_+$ and the anti-twist operator $\sigma_{-\nu}$ comes along with the matrix $\sigma_-$. This is because the insertion of $\Sigma_\nu$ changes the boundary conditions to that of the string joining the D0-brane to the D2-brane. While the insertion of $\Sigma_{-\nu}$ changes the boundary conditions to that of the string joining the D2-brane to the D0-brane. that is a string of opposite orientation. $\lambda_+$ and $\lambda_-$ are complex conjugates of each other. The conformal dimension of the operator $O_1$ is $\nu/2$

As $O_1$ is the bottom component of the the superfield, the operator $O_2$ is obtained from $O_1$ by action of the supercurrent $G_{-1/2}$ on the state $O_1$. This is done in appendix A in (A.4) and (A.5). We get

$$O_2 = \lambda_+ \Lambda_\nu \sigma_+ + \lambda_- \Lambda_{-\nu} \sigma_- \,, \tag{4.5}$$

where

$$\Lambda_\nu = -\frac{i}{\sqrt{2}} \tau_\nu e^{(\nu-1)H} \qquad \Lambda_{-\nu} = -\frac{i}{\sqrt{2}} \tau_{-\nu} e^{-(\nu-1)H} \,. \tag{4.6}$$

The excited twist operators $\tau_\nu$ and $\tau_{-\nu}$ are defined in (A.1). We note that the conformal dimension of $O_2$ is $\nu/2 + 1/2$. The tachyon superfield $\Gamma$ is a world sheet boson. This implies the $O_1$ is a boson and $O_2$ is a world sheet fermion.

### 4.2 The correlation functions

To evaluate the disc partition function up to the the quartic order we will need the two point and four point function of these operators. We evaluate these correlators in the appendix. The two point functions of the twist operators is given by[4]

$$\langle \Sigma_\nu(\tau_1)\Sigma_{-\nu}(\tau_2)\rangle = \left|2\sin\frac{(\tau_1-\tau_2)}{2}\right|^{-\nu}. \tag{4.7}$$

This correlation function is fixed by the dimension of $\Sigma$. Its normalization is fixed to be unity. The two point functions of the excited twist operators $\Lambda$ is given by

$$\langle \Lambda_\nu(\tau_1)\Lambda_{-\nu}(\tau_2)\rangle = -\nu\left|2\sin\frac{(\tau_1-\tau_2)}{2}\right|^{-\nu-1}\Theta(\tau_1-\tau_2). \tag{4.8}$$

The normalization of this correlation function is determined in appendix B. In (4.7) and (4.8) we have written down the correlators on the boundary of the unit disc and for arbitrary time orderings. The theta function appears as the $\Lambda$'s are world sheet fermions.

The following four point functions are needed for the evaluation of the disc partition function.

$$\langle \Sigma_{-\nu}(\tau_1)\Sigma_{+\nu}(\tau_2)\Sigma_{-\nu}(\tau_3)\Sigma_{+\nu}(\tau_4)\rangle = \tau_{12}^{-\nu}\tau_{13}^{+\nu}\tau_{14}^{-\nu}\tau_{23}^{-\nu}\tau_{24}^{+\nu}\tau_{34}^{-\nu}\frac{1}{F(\nu,1-\nu,1;x)}, \tag{4.9}$$

where $x$ is the cross ratio given by

$$x=\frac{\tau_{12}\tau_{34}}{\tau_{13}\tau_{24}} \qquad \text{and} \qquad \tau_{ij}=2\sin\frac{(\tau_i-\tau_j)}{2} \tag{4.10}$$

$F(\nu,1-\nu,1;x)$ is the hypergeometric function defined in (B.7). In (4.9) we have evaluated the correlation function for the time ordering $\tau_1>\tau_2>\tau_3>\tau_4$. We will see in section 5 that we need the four point function in (4.9) at $\nu=1$. At $\nu=1$ the four point function simplifies. It is given by simple Wick contractions.

$$\langle \Sigma_{-\nu}(\tau_1)\Sigma_{+\nu}(\tau_2)\Sigma_{-\nu}(\tau_3)\Sigma_{+\nu}(\tau_4)\rangle|_{\nu=1} = |\tau_{12}\tau_{23}|^{-1}+|\tau_{14}\tau_{23}|^{-1}. \tag{4.11}$$

In (4.11) we have written down the four point function at $\nu=1$ for arbitrary time orderings.

Now we need the four point function of the excited twist operator $\Lambda$. This is given by

$$\langle \Lambda_{-\nu}(\tau_1)\Lambda_{+\nu}(\tau_2)\Lambda_{-\nu}(\tau_3)\Lambda_{+\nu}(\tau_4)\rangle = \tau_{12}^{-2h}\tau_{13}^{2h}\tau_{14}^{-2h}\tau_{23}^{-2h}\tau_{24}^{2h}\tau_{34}^{-2h}G(x). \tag{4.12}$$

[4]Here we have mapped the correlation functions found in appendix B and appendix C to the boundary of the disc from the boundary of the upper half plane.

Here $h = \nu/2 + 1/2$ and $G(x)$ is given (C.7). The correlation function in (4.12) is written down for the time ordering $\tau_1 > \tau_2 > \tau_3 > \tau_4$. As before it is important to note that for $h = 1$ the four point function in (4.12) simplifies to

$$\langle \Lambda_{-\nu}(\tau_1)\Lambda_{+\nu}(\tau_2)\Lambda_{-\nu}(\tau_3)\Lambda_{+\nu}(\tau_4)\rangle|_{\nu=1} = \Theta(\tau_1-\tau_2)\Theta(\tau_3-\tau_4)(\tau_{12}\tau_{34})^2 + \\ +\Theta(\tau_1-\tau_4)\Theta(\tau_2-\tau_3)(\tau_{14}\tau_{23})^2 \,. \quad (4.13)$$

Here we have written the simplified four point function for $\nu = 1$ and for arbitrary time orderings. The $\Theta$ function appears as the $\Lambda$'s are world sheet fermions.

Finally we need the following four point function

$$\langle \Lambda_{-\nu}(\tau_1)\Lambda_{+\nu}(\tau_2)\Sigma_{-\nu}(\tau_3)\Sigma_{+\nu}(\tau_4)\rangle = \tau_{12}^{-2h}\tau_{23}^{\nu}\tau_{14}^{-\nu}\tau_{23}^{-\nu}\tau_{24})^{\nu}\tau_{34}^{-\nu}I(x)\,. \quad (4.14)$$

Where $I(x)$ is given in (B.10) and the four point function in (4.14) is written down for $\tau_1 > \tau_2 > \tau_3 > \tau_4$. Now we need this correlation function with $\nu = 1$. For arbitrary time orderings this is given by

$$\langle \Lambda_{-\nu}(\tau_1)\Lambda_{+\nu}(\tau_2)\Sigma_{-\nu}(\tau_3)\Sigma_{+\nu}(\tau_4)\rangle|_{\nu=1} = \Theta(\tau_3-\tau_4)\tau_{12}^{-2}|\tau_{34}|^{-1}\,. \quad (4.15)$$

Here again the $\Theta$ function appears because the $\Lambda$'s are world sheet fermions. Notice that again for $\nu = 1$ the four point function in (4.14) is determined by simple Wick contractions.

## 5. Evaluation of the disc partition function

In this section we evaluate the disc partition function using the perturbative expansion in (2.7). We will justify this expansion for an almost marginal tachyon operator of the D0-D2 system.

### 5.1 The quadratic term in the disc partition function

The quadratic term $S_2$ in the disc partition function using the renormalization prescription introduced by [20] is given by

$$\begin{aligned} S_2 &= -\langle S_{\text{Boundary}}\rangle \\ &= \lim_{\epsilon\to 0} -\frac{1}{4}\int \frac{d\tau}{2\pi}\langle O_1(\tau)O_1(\tau-\epsilon) + O_2(\tau)\int \frac{1}{2}\Theta(\tau-\tau'-\epsilon)O_2(\tau')d\tau'\rangle\,. \end{aligned} \quad (5.1)$$

This point splitting regularization was motivated by the requirement of a finite one point function in [20]. We now show that the divergences of the first term and the second term in (5.1) cancel leading to a finite term. The first term in (5.1) is given by

$$S_2^1 = -\frac{1}{4}\int \frac{d\tau}{2\pi}\langle O_1(\tau)O_1(\tau-\epsilon)\rangle$$

$$
\begin{aligned}
&= -\frac{1}{4}\int \frac{d\tau}{2\pi}\mathrm{Tr}(\sigma_+\sigma_-)\langle\Sigma_+(\tau)\Sigma_-(\tau-\epsilon)\rangle + \mathrm{Tr}(\sigma_-\sigma_+)\langle\Sigma_-(\tau)\Sigma_+(\tau-\epsilon)\rangle \\
&= -\frac{\lambda_+\lambda_-}{2}\int \frac{d\tau}{2\pi}\frac{1}{(2\sin\frac{\epsilon}{2})^\nu} \\
&= -\frac{\lambda_+\lambda_-}{2}\frac{1}{(2\sin\frac{\epsilon}{2})^\nu}\,. 
\end{aligned} \tag{5.2}
$$

Here we have used the two point function in (4.7). For simplicity we have taken $\epsilon$ to be positive. This can be done without loss of generality. Now the second term in (5.1) is given by

$$
\begin{aligned}
S_2^2 &= -\frac{1}{4}\langle\int \frac{d\tau}{2\pi}O_2(\tau)\int d\tau' \frac{\Theta(\tau-\tau'-\epsilon)}{2}O_2(\tau')\rangle \\
&= \frac{\nu\lambda_+\lambda_-}{4}\int d\tau \frac{\Theta(\tau)\Theta(\tau-\epsilon)}{|2\sin\frac{\tau}{2}|^{2h}} \\
&= \frac{\nu\lambda_+\lambda_-}{4}\left(\int_0^\pi d\tau\frac{1}{(2\sin\frac{\tau}{2})^{2h}} + \int_0^\pi d\tau\frac{\Theta(\tau-\epsilon)}{(2\sin\frac{\tau}{2})^{2h}}\right).
\end{aligned} \tag{5.3}
$$

Where we have taken the traces over the matrices. Substituting $u = \tan(\tau/2)$ and integrating by parts we obtain

$$
S_2^2 = \frac{\nu\lambda_+\lambda_-}{2^{2h+1}}\left(-\frac{4(1-h)}{2h-1}\int_0^\infty du\frac{u^{2-2h}}{(1+u^2)^{2-h}} + \frac{2}{2h-1}\int_0^\infty \frac{du\,\delta(u-\tan\frac{\epsilon}{2})}{u^{2h-1}(1+u^2)^{1-h}}\right). \tag{5.4}
$$

It is important to note that while integrating by parts the boundary terms either cancel or they are identically zero. We expand in $\epsilon$ and retain the singular and finite term in $\epsilon$. We get

$$
S_2^2 = -(1-h)\frac{\pi}{4}\lambda_+\lambda_- + \frac{\nu}{2(2h-1)}\frac{1}{(2\tan\frac{\epsilon}{2})^{2h-1}}\lambda_+\lambda_-\,. \tag{5.5}
$$

We have retained the leading order in $(1-h)$ in the first term in the above expression. Using $2h-1=\nu$ we see that on adding $S_2^1$ and $S_2^2$ see that the divergence in $\epsilon$ cancels and we obtain the following finite term.

$$
S_2 = -(1-h)\frac{\pi}{4}\lambda_+\lambda_-\,. \tag{5.6}
$$

Note that the coefficient of the quadratic term is proportional to the deviation from marginality of the boundary tachyon interaction. Now the validity of the perturbative expansion in (2.7) is clear. We are interested in the stable minimum of the disc partition function. The minimum can be evaluated to an accuracy of $O((1-h)^2)$ if one knows the quartic term in the expansion in (2.7). For this it is sufficient to evaluate the quartic term with $h=1$. This implies $\nu=1$. Furthermore the couplings at the minimum are of the order of $O(1-h)$ which justifies the expansion for an almost marginal operator.

### 5.2 The quartic term in the disc partition function

As we have mentioned above for the accuracy we are interested we can evaluate the quartic term with $\nu = 1$. Thus the four point functions we will require are give in (4.11), (4.13) and (4.15). The quartic term consists of three terms

$$S_4 = \frac{1}{2}(S_4^1 + S_4^2 + S_4^3)\,. \tag{5.7}$$

We will discuss them below. $S_4^1$ is given by

$$S_4^1 = \frac{1}{64\pi^2}\int d\tau d\tau' \langle O_1(\tau) O_1(\tau-\epsilon) O_1(\tau') O_1(\tau'-\epsilon)\rangle\,. \tag{5.8}$$

Substituting the four point function from (4.11), taking the trace and using some straight forward simplifications we obtain

$$\frac{S_4^1}{(\lambda_+\lambda_-)^2} = \frac{1}{32\sin^2\frac{\epsilon}{2}} + \frac{1}{32\pi}\int_0^\pi d\tau \frac{1}{|\sin\frac{\tau-\epsilon}{2}\sin\frac{\tau+\epsilon}{2}|}\,. \tag{5.9}$$

Simplifying further we obtain

$$\frac{S_4^1}{(\lambda_+\lambda_-)^2} = \frac{1}{32\sin^2\frac{\epsilon}{2}} + \frac{1}{16\pi\cos\frac{\epsilon}{2}\sin\frac{\epsilon}{2}}\int_0^{\tan\frac{\epsilon}{2}} du\left(\frac{1}{\tan\frac{\epsilon}{2} - u} + \frac{1}{\tan\frac{\epsilon}{2} + u}\right). \tag{5.10}$$

Let us now focus on the next term $S_4^2$. It is given by

$$\begin{aligned} S_4^2 = \frac{1}{128\pi^2}\Bigg(&\Big\langle \int d\tau_1 O_1(\tau_1) O_1(\tau_1-\epsilon)\int d\tau_2 d\tau' O_2(\tau_2)\Theta(\tau_2-\tau'-\epsilon)O_2(\tau')\Big\rangle + \\ &+ \Big\langle \int d\tau_2 d\tau' O_2(\tau_2)\Theta(\tau_2-\tau'-\epsilon)O_2(\tau')\int d\tau_1 O_1(\tau_1)O_1(\tau_1-\epsilon)\Big\rangle\Bigg)\,. \end{aligned}$$

Using the four point function in (4.13), taking the traces and performing straightforward manipulations we obtain

$$\begin{aligned} \frac{S_4^2}{(\lambda_+\lambda_-)^2} &= -\frac{1}{16}\frac{1}{\sin\frac{\epsilon}{2}}\int d\tau \frac{\Theta(\tau)\Theta(\tau-\epsilon)}{(2\sin\frac{\epsilon}{2})^2} \\ &= -\frac{1}{16}\frac{1}{\sin\frac{\epsilon}{2}\tan\frac{\epsilon}{2}}\,. \end{aligned} \tag{5.11}$$

Finally the third term $S_4^3$ is given by

$$\begin{aligned} S_4^3 = \frac{1}{16}\Big\langle &\int \frac{d\tau_1}{2\pi} d\tau' \frac{\Theta(\tau_1-\tau'-\epsilon)}{2} O_2(\tau_1) O_2(\tau') \times \\ &\times \int \frac{d\tau_2}{2\pi} d\tau'' \frac{\Theta(\tau_2-\tau''-\epsilon)}{2} O_2(\tau_2) O_2(\tau'')\Big\rangle\,. \end{aligned} \tag{5.12}$$

Substituting the correlation function of $O_2$ from (4.15) with $\nu = 1$ and taking the traces and using some straight forward manipulations we obtain

$$\begin{aligned}\frac{S_4^3}{(\lambda_+\lambda_-)^2} &= \frac{1}{32}\frac{1}{(\tan\frac{\epsilon}{2})^2} - \frac{1}{32\pi}\int_0^\pi d\tau \frac{1}{|\tan\frac{\tau+\epsilon}{2}\tan\frac{\tau-\epsilon}{2}|} \\ &= \frac{1}{32}\frac{1}{(\tan\frac{\epsilon}{2})^2} - \frac{1}{16\pi}\frac{1-\tan^2\frac{\epsilon}{2}}{\tan\frac{\epsilon}{2}}\int_0^{\tan\frac{\epsilon}{2}} du\left(\frac{1}{\tan\frac{\epsilon}{2}-u} + \frac{1}{\tan\frac{\epsilon}{2}+u}\right) + \\ &\quad + \frac{1}{32} - \frac{1}{8\pi}\epsilon + \frac{1}{16\pi}\frac{1-\tan^2\frac{\epsilon}{2}}{\tan\frac{\epsilon}{2}}\ln\left(\frac{1+\tan^2\frac{\epsilon}{2}}{1-\tan^2\frac{\epsilon}{2}}\right). \end{aligned} \tag{5.13}$$

Now taking the limit $\epsilon \to 0$ and adding the expressions in (5.10), (5.11) and (5.13) we find that all divergences cancel leaving a finite term

$$S_4 = \frac{1}{2}(S_4^1 + S_4^2 + S_4^3) = \frac{1}{64}(\lambda_+\lambda_-)^2 . \tag{5.14}$$

Putting the quadratic and the quartic terms together we obtain the following expansion for the disc partition function till the quartic order

$$\begin{aligned}\frac{Z[\lambda_+\lambda_-]}{Z[0]} &= 1 - \frac{(1-h)\pi}{4}\lambda_+\lambda_- + \frac{1}{64}(\lambda_+\lambda_-)^2 \\ &= 1 - \frac{1}{8b}\lambda_+\lambda_- + \frac{1}{64}(\lambda_+\lambda_-)^2 . \end{aligned} \tag{5.15}$$

We note that the above expansion of the disc partition function is valid for the tachyon boundary operator which is almost marginal. We have $h \sim 1$ for $b \to \infty$. In the second line of the above equation we have substituted for $h$ in terms of $b$.

## 6. Comparison with the mass defect

According to the proposal of [20] the boundary string field theory action is expected to be the disc partition function. Thus the difference in the value of the stable minimum and the unstable maximum of the disc partition function is expected to reproduce the mass defect involved in tachyon condensation. For the case of the D0-D2 system the difference in the two critical points to $O((1-h)^2)$ is given by

$$Z[(\lambda_+\lambda_-)^*] - Z[0] = -\frac{Z[0]}{4b^2} , \tag{6.1}$$

where $(\lambda_+\lambda_-)^* = 4/b$. $Z(0)$ is twice the mass of the D0-brane. This can be seen as follows. For $b \to -\infty$ we have seen that the system effectively reduces to the D0-brane anti-D0-brane system. From [20] we see that the tachyon potential for the D0-brane anti-D0-brane is given by

$$Z[0]e^{-\frac{(\lambda_+\lambda_-)^2}{4}} , \tag{6.2}$$

where $Z[0]$ is twice the mass of the D0-brane. This reproduces the right mass defect



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for the D0-brane anti-D0-brane system. We can also understand the factor of 2 from the trace of the $2 \times 2$ identity matrix in $Z[0]$. Therefore the difference in the two critical points of the disc partition function is given by

$$Z[(\lambda_+\lambda_-)^*] - Z[0] = -\frac{1}{2\sqrt{2}gb^2}\,. \tag{6.3}$$

We have thus reproduced the leading order term for the mass defect in the formation of the bound state of D0-D2 system given in (3.16) from the disc partition function.

## 7. Conclusions

We have evaluated the disc partition function for the D0-D2 system in the presence of a large B-field using perturbation theory. We have shown that the mass defect calculated from the disc partition function agrees exactly with the expected result in the large B-field limit. This lends support to the recent proposal [20] that the disc partition function is the background independent superstring field theory action. We can compare this result with that obtained using Berkovits' string field theory. The tachyon potential for the D0-D4 system was evaluated at the zeroth level of approximation in [34]. It is easy to extend that result for the D0-D2 system. We see that the zeroth level truncation in Berkovits' string field theory gives only 25% of the expected result for large value of the B-field. This implies that the tachyon in the Berkovits' string field theory mixes with higher level fields. So it seems that the disc partition function is better suited to study the phenomenon of tachyon condensation.

The disc partition function is perhaps undefined for arbitrary boundary operators. In this paper we have shown that one can set up a valid perturbative expansion of the disc partition function for boundary operators which are almost marginal using the D0-D2 system as an example. We showed that the perturbative expansion is valid if one is interested in the value of the disc partition at its critical points. To obtain the mass defects in tachyon condensation the value of the critical points of the disc partition function is sufficient.

In [20] the disc partition function for a world sheet supersymmetric theory was also conjectured to be the boundary entropy [39]. The disc partition function satisfies the condition that it is stationary at fixed points of the renormalization group flow and it takes the right value. It would be interesting to see if it can be shown that the disc partition function for the D0-D2 system decreases along the renormalization group flow set up by the relevant tachyon operator. This would be a tractable problem as the operator can be made almost marginal.[5] This would provide additional evidence for the disc partition function to coincide with the boundary entropy for a world sheet supersymmetric theory.

[5] It is easy to see that the disc partition function at the unstable maximum $\lambda_+ = \lambda_- = 0$ is higher than the stable minimum $(\lambda_+\lambda_-)^*$ from (6.1). The implies the renomalization group flow should drive the system to the stable minimum.

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## A. The excited twist operators

This appendix discuss the definition of the bosonic and fermionic twist operators which are required to construct the tachyon operator $O_1$ and $O_2$. The twist operators are located on the boundary of the world sheet. Thus they are on the real axis when the world sheet is the upper half plane. The insertion of the $\sigma_\nu$ changes the boundary conditions of the string to that of a string joining the D0-brane to the D2-brane. The anti-twist operator $\sigma_{-\nu}$ changes the boundary conditions to that of a string joining the D2-brane to the D0-brane. From the mode expansions in (3.4) the bosonic twist operators $\sigma_\nu$ and $\sigma_{-\nu}$ have the following OPE with the world sheet bosons

$$\begin{aligned}
\partial X^+(z)\sigma_\nu(w) &= \frac{1}{(z-w)^{(1-\nu)}}\tau_\nu(w) & \partial X^-(z)\sigma_\nu(w) &= \frac{1}{(z-w)^{\nu}}\tau'_\nu(w)\\
\partial X^+(z)\sigma_{-\nu}(w) &= \frac{1}{(z-w)^{\nu}}\tau'_{-\nu}(w) & \partial X^-(z)\sigma_{-\nu}(w) &= \frac{1}{(z-w)^{(1-\nu)}}\tau_{-\nu}(w)\\
\bar\partial X^+(\bar z)\sigma_\nu(w) &= -\frac{1}{(\bar z-w)^{1-\nu}}\tau_\nu(w) & \bar\partial X^-(\bar z)\sigma_\nu(w) &= -\frac{1}{(\bar z-w)^{\nu}}\tau'_\nu(w)\\
\bar\partial X^+(\bar z)\sigma_{-\nu}(w) &= -\frac{1}{(\bar z-w)^{\nu}}\tau'_{-\nu}(w) & \bar\partial X^-(\bar z)\sigma_{-\nu}(w) &= -\frac{1}{(\bar z-w)^{(1-\nu)}}\tau_{-\nu}(w)\,.
\end{aligned} \tag{A.1}$$

Here $w$ is a point on the real axis. The $\tau$'s are the excited twist operators. To construct the fermionic twist we first bosonize the fermions by defining

$$\psi^+ = i\sqrt{2}e^{iH} \qquad \psi^- = i\sqrt{2}e^{-iH}\,, \tag{A.2}$$

where $H$ is a free boson. The fermionic twist operator is given by $e^{i\nu H}$ and the anti-twist operator is given by $e^{-i\nu H}$. The OPE of these twist operators with the fermions is given by

$$\begin{aligned}
\psi^+(z)e^{i\nu H(w)} &= i\sqrt{2}(z-w)^\nu e^{i(\nu+1)H(w)} & \psi^-(z)e^{i\nu H(w)} &= \frac{i\sqrt{2}}{(z-w)^\nu}e^{-i(1-\nu)H(w)}\\
\psi^+(z)e^{-i\nu H(w)} &= \frac{i\sqrt{2}}{(z-w)^\nu}e^{i(1-\nu)H(w)} & \psi^-(z)e^{-i\nu H(w)} &= i\sqrt{2}(z-w)^\nu e^{-i(\nu+1)H(w)}.
\end{aligned} \tag{A.3}$$

Now we have all the ingredients necessary for defining the twist operator $\Sigma_\nu$ and $\Sigma_{-\nu}$ given in (4.1). Using the OPE's in (A.1) and (A.3) we find that the top component

$\Lambda_\nu$ of the twist field $\Sigma_\nu$ is given by the following OPE

$$G(z)\Sigma_\nu(w) = \frac{1}{2}\frac{\Lambda_\nu(w)}{(z-w)}\,. \tag{A.4}$$

Similarly $\Lambda_{-\nu}$ is given by

$$G(z)\Sigma_{-\nu}(w) = \frac{1}{2}\frac{\Lambda_{-\nu}}{(z-w)}\,. \tag{A.5}$$

Here $G(z)$ is given by

$$G(z) = -\frac{1}{4}(\psi^+\partial X^- + \psi^-\partial X^+)\,. \tag{A.6}$$

Here we have defined the supercurrent $G(z)$ only for the relevant two coordinates, $X^+$ and $X^-$. Thus the top component of the twist field is given by

$$\Lambda_\nu = -\frac{i}{\sqrt{2}}\tau_\nu e^{-i(1-\nu)H} \qquad \Lambda_{-\nu} = -\frac{i}{\sqrt{2}}\tau_\nu e^{i(1-\nu)H} \tag{A.7}$$

# B. Normalization of the two point function the twist operators

In this section we discuss the normalization of the two point function given in (4.7) and (4.8). We first normalize the two point function of the bosonic twist operator to be

$$\langle\sigma_\nu(z)\Sigma_{-\nu}(w)\rangle = \frac{1}{(z-w)^{\nu(1-\nu)}}\,. \tag{B.1}$$

This normalization fixes the two point function of the twist operator $\Sigma_\nu$ to be

$$\langle\Sigma_\nu(z)\Sigma_{-\nu}(w)\rangle = \frac{1}{(z-w)^{\nu}}\,. \tag{B.2}$$

Now we fix the normalization of the two point function of the excited twist operators. Consider the correlator

$$g(z,w) = \frac{\left\langle -\frac{1}{2}\partial X^+(z)\partial X^-(w)\sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\right\rangle}{\langle\sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\rangle}\,. \tag{B.3}$$

Taking the limit

$$\lim_{z\to z_2 w\to z_1}\left((z-z_2)^{(1-\nu)}(w-z_1)^{(1-\nu)}g(z,w)\right) = -\frac{1}{2}\frac{\langle\tau_{-\nu}(z_1)\tau_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\rangle}{\langle\sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\rangle}\,, \tag{B.4}$$

we can obtain the four point function $\langle\tau_{-\nu}(z_1)\tau_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\rangle$. From this we will determine the normalization of the two point functions of $\sigma_\nu$. Evaluation of $g(z,w)$ was reviewed and $\langle\sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\rangle$ in [34]. Here we just state the result.

$$g(z,w) = \omega_\nu(z)\omega_{1-\nu}(w)\Bigg(\nu\frac{(z-z_1)(z-z_3)(w-z_2)(w-z_4)}{(z-w)^2} + (1-\nu)\times \\ \times\frac{(z-z_2)(z-z_4)(w-z_1)(w-z_3)}{(z-w)^2} + A(z_1,z_2,z_3,z_4)\Bigg).$$

Where $\omega_\nu(z)$ is given by

$$\omega_\nu(z) = \frac{1}{[(z-z_1)(z-z_3)]^\nu}\frac{1}{[(z-z_2)(z-z_4)]^{1-\nu}} \tag{B.5}$$

and

$$A(0,x,1,z_4) = -z_4x(1-x)\frac{d}{dx}\ln F(\nu,1-\nu,1;x) \tag{B.6}$$

here $z_4$ stands for $\infty$. $F(\nu,1-\nu,1;x)$ is the hypergeometric function whose integral representation is given by

$$F(\nu,1-\nu,1;x) = \frac{1}{\pi}\sin(\pi\nu)\int_0^1 dy\frac{1}{y^\nu(1-y)^{1-\nu}(1-xy)^\nu}\,. \tag{B.7}$$

The four point function of the bosonic twists are given by

$$\langle\sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_\nu(z_4)\rangle = z_{12}^{-2h'}z_{13}^{2h'}z_{14}^{-2h'}z_{23}^{-2h'}z_{24}^{2h'}z_{34}^{-2h'}\frac{1}{F(\nu,1-\nu,1;x)}\,, \tag{B.8}$$

where $z_{ij} = z_i - z_j$, $h' = \nu(1-\nu)/2$ and $x = (z_{12}z_{34})/(z_{13}z_{24})$. Using (B.4) and (B.8) we find

$$\langle\tau_{-\nu}(0)\tau_\nu(x)\sigma_{-\nu}(1)\sigma_\nu(z_4)\rangle = \frac{1}{x^{\nu(3-\nu)}(1-x)^{\nu(2-\nu)}z_4^{\nu(1-\nu)}}I(x)\,, \tag{B.9}$$

where $I(x)$ is given by

$$I(x) = \frac{1}{F(\nu,1-\nu,1;x)}\left(\nu(1-x) + x(1-x)\frac{d}{dx}\ln F(\nu,1-\nu,1;x)\right). \tag{B.10}$$

It is easy to see that using (B.1) and (B.9) we obtain the following normalization for the two point functions of the excited twist operators.

$$\langle\tau_{-\nu}(z_1)\tau_\nu(z_2)\rangle = \frac{2\nu}{(z_1-z_2)^{\nu(3-\nu)}}\,. \tag{B.11}$$

Using the above equation we see that

$$\langle\Lambda_\nu(z_1)\Lambda_{-\nu}(z_2)\rangle = -\frac{\nu}{(z_1-z_2)^{\nu+1}}\,. \tag{B.12}$$



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## C. The four point function of the excited twist operators

In this section we will focus on the evaluation of the four point function involving the excited twist operators. We first evaluate the four point function of the excited twist operators given by $\langle \tau_{-\nu}(z_1)\tau_\nu(z_2)\tau_{-\nu}(z_3)\tau_\nu(z_4)\rangle$. To do this first consider the auxiliary correlator

$$f(z,w,z',w') = \frac{\langle E(z,w,z',w')\sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_{-\nu}(z_4)\rangle}{\langle \sigma_{-\nu}(z_1)\sigma_\nu(z_2)\sigma_{-\nu}(z_3)\sigma_{-\nu}(z_4)\rangle}\,, \tag{C.1}$$

where $E(z,w,z',w')$ is given by

$$E(z,w,z,w') = \left(-\frac{1}{2}\partial X^+(z)\partial X(w)\right)\left(-\frac{1}{2}\partial X^+(z')\partial X(w')\right). \tag{C.2}$$

The function $f(z,w,z',w')$ can be determined by singularity structure and monodromy conditions. The following form for $f(z,w,z',w')$ has the required singularity structure.

$$\begin{aligned} f(z,w,z',w') = {} & \omega_\nu(z)\omega_{1-\nu}(w)\omega_\nu(z')\omega_{1-\nu}(w')\,\times \\ & \times\left[e(z,w)e(z',w') + e(z',w)e(z,w') + B(z_1,z_2,z_3,z_4)\right]. \end{aligned} \tag{C.3}$$

The function $e(z,w)$ is given by

$$\begin{aligned} e(z,w) = {} & \left(\nu\frac{(z-z_1)(z-z_3)(w-z_2)(w-z_4)}{(z-w)^2} + (1-\nu)\,\times\right. \\ & \left.\times\,\frac{(z-z_2)(z-z_4)(w-z_1)(w-z_3)}{(z-w)^2} + A(z_1,z_2,z_3,z_4)\right). \end{aligned} \tag{C.4}$$

Here $B(z_1,z_2,z_3,z_4)$ is a function which can be fixed by monodromy conditions. That is, the change in $X^+$ from going to one Dirichlet boundary to another is zero. Using this input we find that $B(z_1,z_2,z_3,z_4)$ is zero. Now taking the limit

$$\lim_{z\to z_2, w\to z_1, z'\to z_4, w'\to z_3} \left((z-z_2)(w-z_1)(z'-z_4)(w'-z_3)\right)^{1-\nu} f(z,w,z',w') \tag{C.5}$$

we can extract out the required correlation function. We find

$$\langle \tau_{-\nu}(z_1)\tau_\nu(z_2)\tau_{-\nu}(z_3)\tau_\nu(z_4)\rangle = \frac{4}{z_{12}^{\nu(3-\nu)} z_{13}^{-\nu(3-\nu)} z_{14}^{\nu(3-\nu)} z_{23}^{\nu(3-\nu)} z_{24}^{-\nu(3-\nu)} z_{34}^{\nu(3-\nu)}} G(x)\,, \tag{C.6}$$

where $G(x)$ is given by

$$\begin{aligned} G(x) = {} & \frac{1}{F(\nu,1-\nu,1,x)}\left((1-x)^2\left[\nu + x\frac{d}{dx}\ln F(\nu,1-\nu,1,x)\right]^2 + \right. \\ & \left. + x^2\left[\nu + (1-x)\frac{d}{dx}\ln F(\nu,1-\nu,1,x)\right]^2\right). \end{aligned} \tag{C.7}$$

Putting the bosonic and the fermionic twists together one obtains

$$\langle \Lambda_{-\nu}(z_1)\Lambda_{\nu}(z_2)\Lambda_{-\nu}(z_3)\Lambda_{\nu}(z_4)\rangle = z_{12}^{-(\nu+1)} z_{13}^{\nu+1} z_{14}^{-(\nu+1)} z_{23}^{-(\nu+1)} z_{24}^{(\nu+1)} z_{34}^{-(\nu+1)} G(x)\,. \tag{C.8}$$

Now using (B.9) and the definition of the $\Lambda$ twist operators we find

$$\langle \Lambda_{-\nu}(z_1)\Lambda_{\nu}(z_2)\Lambda_{-\nu}(z_3)\Lambda_{\nu}(z_4)\rangle = z_{12}^{-2h} z_{l3}^{\nu} z_{l4}^{-\nu} z_{23}^{-\nu} z_{24}^{\nu} z_{34}^{-\nu} I(x)\,. \tag{C.9}$$

This completes the derivation of the required four point functions.

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